Towards beliefs on proof in mathematics: a survey of future elementary school teachers in Switzerland

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This paper analyzes the beliefs of proof among first-year bachelor's students, future primary school teachers, in the context of current thinking in proof research. Using a questionnaire tailored to this audience, we study the functions of proof that they recognize, as well as their criteria for evaluating the rigor and validity of different types of proof. The results show a clear gap between their ability to identify intellectual, deductive proof as rigorous and the empirical or perceptual approaches relevant to pragmatic proof that they spontaneously mobilize. Explanatory and verification functions dominate, while more social dimensions appear only marginally. These results highlight the need to reinforce, in initial training, the active and diversified appropriation of the activity of proof.

Keywords: belief, proof, primary school teacher training, mathematical practice

INTRODUCTION

Proof lies at the heart of mathematical activity, yet persistent difficulties in learning and teaching it remain across grade levels and countries. Mathematics education research has highlighted a significant discontinuity between secondary and tertiary education internationally (e.g. Gueudet, 2008; Selden et al., 2010) and teacher education (both pre-service and in-service) often appears insufficient or inadequate. At the international level, few studies have examined teachers' practices or their conceptions (or their beliefs) of proof and its teaching (e.g. Mariotti et al. 2018; Stylianides, 2016). Our work builds on discussions within the international mathematics education community, particularly following Ouvrier-Buffet's (2023) collaborative proposal at CERME 13 concerning undergraduate students' conceptions of proof elicited through questionnaires. This issue has already been investigated with freshmen in Belgium and pre-service teachers in France (De Vleeschouwer & Ouvrier-Buffet, 2024). In this paper, through a questionnaire adapted from previous similar studies (e.g. Healy & Hoyles, 1998; Stylianou, 2015; Ouvrier-Buffet, 2023), we focus on the belief of proof, – in accordance with Beswick (2005) who "refer to anything that an individual regards as true" (ibid., p. 39) –, in a hardly investigated population: first-year Bachelor students entering primary teacher education in Switzerland (who have not taught yet). The instrument includes analyses of multi-proof items, questions about student's school experiences with proof in mathematics classes, and questions about the functions of proof. Our research questions are: *What beliefs of proof do prospective primary teachers display when examining different mathematical proofs? Which functions of proof do they identify in mathematics, and what do their stated beliefs* *relate to the mathematical practice of proof they experienced during their previous schooling?*

THEORETICAL BACKGROUND AND DIMENSIONS OF PROOF ADDRESSED BY THE QUESTIONNAIRE

Ouvrier-Buffet (2023) developed a questionnaire based on a bibliographical study to determine how researchers in mathematics education have already investigated the undergraduate students' conceptions of proof (based on their statements, assessment and writing about proofs by comparing what they say with what they do). The study highlights the interest of Balacheff's (1988) theoretical framework and notes that few items in the literature explore students' judgements of given proofs and their global perceptions and conceptions of proof. Ouvrier-Buffet's (2023) proposal selects mathematical contents where mastering the involved mathematical concepts is minimised and chooses mathematical problems outside of formalism (so that the formalism is not an obstacle) and outside of curricula (to avoid obstacles or ready-made results and processes). Based on results in the literature about students' difficulties (mainly the significant students' difficulties even with deductive short proofs), our questionnaire explores beliefs (Beswick, 2005) focusing on "basic" components of deductive proofs, reusing some items and enriching previous surveys. In particular, we focus on the use of the instances of a mathematical statement, the evaluation of written proofs with multiple choice questions (as Healy & Hoyles 1998 and Stylianou et al. 2015, but with new features and mathematical domains), and the exploration of the students' global beliefs of proof (the way they think about proof) with open questions leading to kinds of proofs, functions of proofs (e.g. Hanna, 2000), and declarative students' tools to write/read/evaluate proofs. As a theoretical background, in order to justify the proofs we submit to the students and to analyse their answers, we use Balacheff (1988)'s characterisation of arguments (mainly naïve empiricism, generic argument and intellectual proof) and we play on the fact that a proof can be written (completely or partially) in a natural language, in a formal language, or can be visual.

METHODOLOGY

In response to Ouvrier-Buffet's (2023) call to investigate university students' conceptions of proof, we adapted the initial questionnaire (ibid.) for students enrolled in primary teacher education at HEP-VS. Drawing on Healy and Hoyles (1998), Stylianou et al. (2015) and Ouvrier-Buffet (2023), we designed a questionnaire containing problems aligned with mathematical content familiar to these students. The instrument comprises five parts: the first two administered in a paper-and-pencil format (40 min.) and the remaining three digitally (40 min.), enabling control over backtracking effects and precise measurement of response times.

- Part I: General beliefs of mathematics and mathematical activity (10 min)
- Part II: Instantiation of known theorems or formulas (30 min)
- Part III: Multiple-proof statement (30 min)

- Part IV: School experiences with proof and perceived functions of proof (10 min)
- Part V: Demographic info: gender, age, prior experience, nationality (2 min).

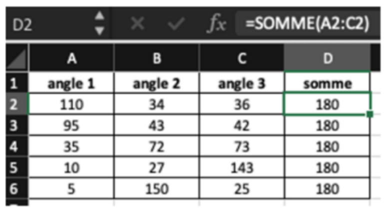
This questionnaire was administered at the beginning of the first year of the primary education program (Bachelor's degree) at HEP-VS. It involved 30 volunteer students with a wide variety of backgrounds. Completing the questionnaire took approximately one hour, including both the paper-and-pencil and digital sections.

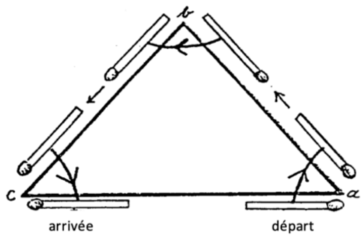
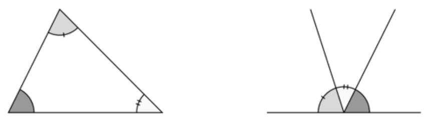
In this paper, we will focus primarily on Parts III and IV. Part III proposes to study proofs for two multi-proof statements, one in the field of arithmetic and the other in the field of geometry.

- Statement 1 – The product of three consecutive numbers is a multiple of 6.
- Statement 2 – The sum of the angles of a triangle is a straight angle.

For each statement, four possible answers are provided, and students are asked to check one of them : *Which answer is nearest to what you would have done?* or *Which answer would you choose to explain the "solution" to your classmates?* or *Which answer is the most "thorough" for you?* or *What is/are the correct proof(s) for you?* or *What is/are the incomplete proof(s) in your opinion?*

Below are examples of proof proposed for each of the statements (Table 1).

	Statement 1	**Statement 2**
Answer A	A number that has 2 and 3 for factors is a multiple of 6. If I have three consecutive numbers, one will be congruent to 0 modulo 3, so it will be a multiple of 3. Also, at least one number will be even and all even numbers are multiples of 2. So, if I multiply three consecutive numbers together, the answer must have at least one factor of 3 and one factor of 2. I can conclude that the product of three consecutive numbers is a multiple of 6.	By measuring the angles of various kinds of triangles and writing them in a spreadsheet, I have:  D2 =SOMME(A2:C2) angle 1 / angle 2 / angle 3 / somme 110 / 34 / 36 / 180 95 / 43 / 42 / 180 35 / 72 / 73 / 180 10 / 27 / 143 / 180 5 / 150 / 25 / 180 Then, I deduce that the sum of the angles of a triangle is equal to 180° (straight angle).
Answer B	$1 \times 2 \times 3 = 6$ $2 \times 3 \times 4 = 24 = 6 \times 4$ $4 \times 5 \times 6 = 120 = 6 \times 20$ $25 \times 26 \times 27 = 17550 = 6 \times 2925$ I can conclude that the product of three consecutive numbers is a multiple of 6.	Let ABC be any triangle. The angles are marked on the figure. A line parallel to (BC) is drawn through A: $p+c+r = 180°$ (straight angle) The alternate interior angles are equal: we deduce the following equalities

		$p=a$ et $r=b$. Thus: $a+b+c=p+c+r$ The sum of the angles of a triangle is equal to 180° (straight angle).
Answer C	Let n be a natural integer. $n \times (n + 1) \times (n + 2) = (n^2 + n) \times (n + 2)$ $= n^3+3n^2 + 2n$ The factor coefficients of the powers of n are: 1+3+2=6 I can conclude that the product of three consecutive numbers is a multiple of 6.	Observe how the matchstick moves around the triangle:  At the end, the matchstick is in the opposite position to the start. During this displacement, and by turning through the three angles of the triangle, the matchstick has made a half-turn. The sum of the angles of a triangle is therefore equal to 180° (straight angle).
Answer D	For three consecutive numbers, let P be the first of them. P is either – even: there exists an integer a such that $P = 2a$ by definition of an even number, – or odd: there exists an integer b such that $P = 2b-1$ by definition of an odd number. If P is even, then the product of three consecutive numbers is: $2a\times(2a+1)\times(2a+2)$: it is a multiple of 2. –If a is a multiple of 3, the proof is done. – If a is not a multiple of 3, then 2a is not a multiple of 3, but, in this case, $(2a+1)$ or $(2a+2)$ is a multiple of 3 and the proof is done. If P is odd, then the product of three consecutive numbers is: $(2b-1)\times 2b\times(2b+1)$: it is a multiple of 2.	We cut out the angles and place them side by side: we get a straight line. The sum of the angles of a triangle is therefore equal to 180° (flat angle). 

	– If *b* is a multiple of 3, the proof is done. – If *b* is not a multiple of 3, then *2b* is not a multiple of 3, but, in this case, *(2b-1)* or (*2b+1)* is a multiple of 3 and the proof is done. I can conclude that the product of three consecutive numbers is a multiple of 6.	

Table 1: Different proofs proposed for statements 1 and 2.

Part IV examines students' beliefs about the role of proof in mathematics and how they encountered proof in their own schooling. Students were asked to indicate the frequency with which they experienced specific classroom situations using two 5-point Likert scales.

- Situation 1. In math classes, proofs were produced by the teacher alone.
- Situation 2. In math classes, I constructed proofs with my classmates.

For these first two situations, the options were: *very often*, *often*, *sometimes*, *rarely*, and *never*. For the next three situations, students selected from: *strongly agree*, *often agree*, *agree*, *neither agree nor disagree*, *disagree*, and *strongly disagree*.

- Situation 3. Proofs were a central focus of math lessons.
- Situation 4. Constructing proofs is fundamental in mathematics.
- Situation 5. One can study mathematics without being skilled at constructing proofs.

The final question in this section addressed the functions of proof. Students indicated what they believe mathematical proofs are used for, choosing among several options: *explaining*, *discovering*, *convincing*, *communicating*, *verifying*, *formalizing*, and *other* (with specification). To analyse responses from Parts III and IV and address our research question, we conducted a brief *a priori* analysis of the proofs proposed in Part III. This enabled us to formulate hypotheses that were subsequently confirmed or refuted using data from Part IV. The following section presents this analysis and our main results.

ANALYSIS AND RESULTS

Table 2 summarises the *a priori* analysis of the proposed proofs for Statements 1 and 2. Our analysis draws primarily on Balacheff's (1988) levels of proof, the level of language used (formal vs. natural), and the validity of the proof (correct vs. incorrect).

	Statement 1	**Statement 2**
Answer A	Natural language, arithmetic register, correct proof.	Naive empiricism / crucial experiment limit; graphical and numerical register, incorrect proof:

		empirical measurement not generalizable.
Answer B	Naive empiricism and crucial experience, excessive generalization based on numerical data from calculated examples; incomplete proof: only a few values have been processed.	Intellectual proof, formal language, alternating between natural and algebraic-symbolic registers. Correct proof.
Answer C	Intellectual proof, formalized language belonging to the algebraic register; incorrect use of the existence theorem: the sum of the coefficients has no relation to the divisibility of the product. Invalid reasoning.	Informal but formalizable and generic visual proof, manipulation, correct. Mathematical theory not explained, not formalized, outside the conventional and institutional framework (French and Swiss). (This example comes from a Hungarian textbook/course).
Answer D	Intellectual proof, natural language, completeness of cases and disjunction of cases, correct proof.	Visual proof, a form of naive empiricism, incomplete. Manipulation of cut-outs (common in textbooks/resources)

Table 2: Brief preliminary analysis of the various proofs presented in the statements

The average time spent on Part III was 6 minutes and 59 seconds, far below the 30 minutes allocated. This strongly suggests that students did not engage in an actual proving process for Statements 1 and 2, but instead responded intuitively.

Table 3 summarises students' responses in Part III. Overall, the results reveal predominantly empirical beliefs of proof. For both statements, the responses selected as closest to what students would do spontaneously are largely empirical or inductive. For Statement 1, example-based proofs (B: 10 responses) and superficial algebraic treatments (C: 10) prevail. For Statement 2, measurement-based (A: 14) or manipulation-based approaches (D: 11) dominate. This reliance on empirical approaches is confirmed when students choose a proof to explain to classmates: example-based reasoning is widely selected in arithmetic (B: 15), and manipulation-based reasoning in geometry (D: 19). These choices reveal a strong dependence on perceptual or numerical registers, at the expense of deductive reasoning.

However, when students judge rigor or validity, their choices shift markedly toward deductive proofs—those corresponding to Balacheff's (1988) category of intellectual proofs. For Statement 1, the formal proof (D: 17 responses) is considered the most rigorous; for Statement 2, the Euclidean proof (B: 19) is preferred. Proofs judged correct also predominantly rely on structured deductive reasoning, whether arithmetic (D: 20) or geometric (B: 18; D: 23). Conversely, proofs reflecting naïve empiricism are generally identified as incomplete (B: 22 for Statement 1; C: 21 for Statement 2), indicating students' awareness of the limits of perception or isolated examples for generalisation. This suggests that students do not attribute a genuinely explanatory function to proof: examples may help explain, but are recognised as insufficient.

Overall, students display a tension between their spontaneous practice – rooted in empiricism – and their ability to recognise the institutional expectations of proof centered on deduction and explicit use of properties and theorems. This discrepancy may indicate an intermediate stage in the development of their beliefs of proof: they appear to understand what constitutes a proof, yet do not spontaneously mobilise it.

Table 3 presents the detailed results. Each pair (a; b) indicates the number of responses for Statement 1 (a) and Statement 2 (b).

	A	**B**	**C**	**D**
Which answer is nearest to what you would have done?	(7 ; 14)	(10 ; 4)	(10 ; 0)	(2 ; 11)
Which answer would you choose to explain the "solution" to your classmates?	(7 ; 6)	(15 ; 1)	(2 ; 3)	(5 ;19)
Which answer is the most "thorough" for you?	(0 ; 5)	(2 ; 19)	(10 ; 0)	(17 ; 5)
What is/are the correct proof(s) for you?	(16 ; 14)	(11 ; 18)	(20 ; 8)	(20 ; 23)
What is/are the incomplete proof(s) in your opinion?	(14 ; 12)	(22 ; 6)	(5 ; 21)	(2 ; 7)

Table 3: Results of Part III regarding responses to questions on statements 1 and 2.

The results of the first part of Section IV, summarised in Table 4, indicate that most students experienced proof as a predominantly lecture-based—or even monumentalised—activity (Situation 1). Fifteen students reported that proofs were produced “very often” by the teacher and eight “often,” whereas only one student selected “rarely” or “never.” Conversely, collaborative proof construction (Situation 2) appears uncommon: 17 students reported experiencing it “rarely” or “never,” and none selected “very often.” Proof is also not perceived as central to mathematics instruction (Situation 3): only seven students “strongly agree” or “agree,” while 23 express neutrality or disagreement.

Yet, despite limited exposure, students overwhelmingly acknowledge the importance of proof (Situation 4): 24 “strongly agree” or “agree” that constructing proofs is fundamental in mathematics. Regarding whether one can study mathematics without being proficient in proof (Situation 5), opinions vary, but a majority (14 “disagree” or “strongly disagree”) consider proof competence necessary. Overall, these results highlight a tension between mathematical practices of classroom centered on teacher-led demonstration and students’ recognition of the pivotal role of proof in mathematical activity.

	Very often	Often	Sometimes	Rarely	Never
Situation 1	15	8	5	1	1
Situation 2	0	4	9	12	5
	Totally agree	**Agree**	**Neither disagreeing nor agreeing**	**Disagree**	**Strongly disagree**
Situation 3	2	5	10	9	4
Situation 4	11	13	2	3	1
Situation 5	1	10	5	13	1

Table 4: First result of Part IV on students' perceptions of proof practices.

The second part of Section IV focuses on students' views of the functions of proof. As shown in Table 5, most students attribute explanatory (93%) and validating (90%) functions to proof, confirming a belief centered on clarification and verification. This finding refutes the hypothesis, suggested after Part III, that students do not see proof as explanatory. The function of *convincing* is also widely recognised (73%). In contrast, more discursive or structural functions such as *communicating* (30%) and *formalising* (43%) are less represented. Only 16% associate proof with *discovering*, suggesting a limited belief of proof as a means of exploration.

Among the four students who selected *other*, two specified additional roles (*confronting* and *reproducibility*). These responses suggest that proof is seen both as a tool for testing and comparing assertions or lines of reasoning, and as a process whose reproducibility ensures that results can be independently verified by any member of the mathematical community.

Function of proof	Number of responses	Percentage of students
Explain	28	93%
Discover	5	16%
Convince	22	73%
Communicate	9	30%
Verify	27	90%
Formalize	13	43%
Other	4	13%

Table 5: Second result of Part IV on the role of proof (function) by students.

DISCUSSION

Taken together, the results reveal a marked discrepancy between, on the one hand, the institutional belief of proof that students are able to recognise and, on the other, the mathematical practices they actually use or have encountered during their schooling.

In Part III, students clearly identify deductive proofs as the most rigorous and valid even when they are mathematically incorrect (e.g., proof D for Statement 1 with 17 responses, and proof B for Statement 2 with 19). However, when asked which approach they would adopt themselves or select to explain a solution, they overwhelmingly favour empirical, manipulative, or example-based strategies (proofs B and C for Statement 1; A and D for Statement 2). This tension suggests that while students possess a conceptual understanding of the criteria for proof validity, these criteria are not spontaneously mobilised in their own proving or explanatory practices.

The results from Part IV help illuminate this discrepancy. A majority of students reported that proofs were *very often* or *often* produced solely by the teacher (23 responses), whereas collaborative construction of proofs was described as rare or absent (17 responses). Likewise, proof is not perceived as the central activity in mathematics lessons, despite strong agreement that constructing proofs is fundamental (24 students *agree* or *strongly agree*). This indicates a learning environment in which proof is valued in principle but seldom practised, which may account for students' reliance on empirical approaches despite their recognition of the epistemic superiority of deductive reasoning.

Moreover, students' identification of the functions of proof (primarily *explanation*, 93%, and *verification*, 90%) reflects a belief of proof oriented toward clarification and validation, while communicative functions appear less prominent. Notably, the function of *verification* is understood by many students as checking examples rather than engaging in the semantic or syntactic evaluation of a proof. This gap between theoretical understanding, actual practices of proof, and institutional expectations constitutes a significant educational challenge, underscoring the need for instructional approaches that foster the effective production of proofs and a broader repertoire of justification methods.

CONCLUSION AND RESEARCH PERSPECTIVES

The results reveal a clear discrepancy between students' stated beliefs (according to which they correctly identify deductive proof, particularly syntactic and formal forms, as the most rigorous), and their actual practices of proof, which remain largely grounded in empirical or perceptual approaches. This misalignment can be partly attributed to school experiences in which proofs are predominantly presented by the teacher and rarely socially constructed. Although students acknowledge the fundamental role of proof, they prioritise only certain functions (*explaining*, *verifying*, *convincing*), while its social dimensions appear insufficiently recognised. For instance, functions such as *discovering* or *communicating* are perceived in a more superficial manner, and more advanced functions (such as confronting ideas or ensuring reproducibility) emerge only marginally.

These findings call for further research on teaching approaches that foster active student engagement in the production and discussion of proofs. They also highlight the need to develop a shared knowledge base concerning proof and proof processes, as well as to reflect on the institutionalisation of this knowledge: What exactly should be institutionalised? Through which pedagogical means? And under what conditions? As a continuation of this exploratory work, we have slightly revised the questionnaire — modifying certain modalities — in order to administer it to a larger sample of students across different French-speaking contexts (France, Belgium, Switzerland).

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