MODEL FOR RETRANSCRIBING AND ANALYSING THE MATHEMATICAL ACTIVITY OF SUBJECTS IN PROBLEM SOLVING

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In this paper we present a model for tracking the activity traces of individuals placed in autonomous problem solving. In order to collect as many traces as possible, we use a collection method based on audiovisual recordings. This model is presented in three steps. The first step concerns the choice of observables: typology and choice of elementary actions and possible strategies. A second one at the level of the processing of the corpus of data in connection with the retranscription and its summaries. This second step is devoid of interpretation, it is in fact completely deterministic and allows the replay of each of the actions performed by the subjects. The third and final step consists in analyzing this retranscription by identifying sequences of actions that are similar or close to the strategies identified.

INTRODUCTION

A great deal of international research in mathematics education uses audiovisual recording tools to collect data in and beyond the classroom. Audiovisual data collected (video for instance) makes it possible to trace a part of reality, even if it is subject to several biases in collection and interpretation. Moreover, compared with other collection methods, audiovisual data allows the scene to be replayed as many times as necessary so that the analysis can be objective (Da Ronch, 2022). Thus, the aim of this paper is therefore to propose a model for retranscribing and analyzing audiovisual recordings in order to identify the mathematical activity of the subjects. In particular, when subjects are placed in an autonomous situation of problem-solving. Furthermore, the model developed in this document is close to the one used in the kTBS4LA (kernel for Trace Based Systems for Learning Analytics) platform developed by TWEAK[1] team in the "Laboratoire d'informatique en Image et Systèmes d'information (LIRIS)" at the University Claude Bernard Lyon (France). The kTBS4LA platform is designed to collect, analyze, visualize and interpret data (Casado et al., 2017). Here, our model, named **ORA**, will be presented in three steps: **O**bservable, **R**etranscription and **A**nalysis (see e.g., Da Ronch, 2022; Da Ronch et al., 2023). For each step, we will attempt to provide answers to the following questions: What are the choices of observables? How can we retranscribe the subjects' activity? How can we analyze these retranscriptions?

DESCRIPTION OF THE ORA-MODEL: CHOICES OF THE OBSERVABLES

In the choice of observables, we first need to identify elementary actions noted a_i. Among these a_i-actions, we retain a subset $\mathbb{A}$ for the epistemological and didactical reasons that is determined thanks to our knowledges on the problem, given by the mathematical analysis. We impose that the subset $\mathbb{A}$ be finite because the corpus of data is also finite. The a_i-actions are carried out on the environment of the situation named *milieu* (Brousseau, 1997). We categorize the *milieu* into functional zones (Da

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[1] TWEAK: Traces, Web, Education, Adaptation, Knowledge. https://liris.cnrs.fr/equipe/tweak

Ronch, 2022). The Work Zone ($\mathcal{W}_\mathcal{Z}$) is the main zone of the environment where the subject will act to respond to the problem. Actions in this zone are therefore actions aimed at resolving the problem. Then, the Tooling Zone ($\mathcal{T}_\mathcal{Z}$) is the area of the environment that provides the subject(s) with the objects needed to develop problem-solving actions. Finally, the Information Zone ($\mathcal{I}_\mathcal{Z}$) is the area of the environment that communicates information about the problem: rules of the game, instructions, examples, *etc*.

Typology of actions

Now, we can classify the a_i-actions according to these functional zones. The $\mathcal{W}$-actions which are actions that modify the $\mathcal{W}_\mathcal{Z}$.The $\mathcal{T}$-actions that interact with the $\mathcal{T}_\mathcal{Z}$. The $\mathcal{I}$-actions that allow access to the $\mathcal{I}_\mathcal{Z}$. Finally, there are the $\mathcal{S}_O$-actions, which are actions arising from social interactions involving exchanges between peers or even oneself. They can be actions in oral language or actions that involve communicating in sign language (gestures).

Strategies for solving the problem

The choice of our finite set of the $\mathcal{W}$-actions, $\mathcal{T}$-actions, $\mathcal{I}$-actions and $\mathcal{S}_O$-actions is conditioned by our knowledge of how to solve the given problem. The mathematical analysis of the problem enables us to identify the strategies for solving it. Since the latter is theoretically "perfect", it gives us exhaustive access to all the strategies. We characterize a strategy as an ordered concatenation of the actions (a_i) carried out on instantiated objects (o_j). A strategy is therefore a word that takes take the form of an ordered sequence noted $a_i o_j$.

DESCRIPTION OF THE ORA-MODEL: RETRANSCRIPTION OF EXPERIENCE

Encoding of a_i-actions in a symbolic format

To make our transcription intelligible, we bijectively encode the actions in an intelligible symbolic format taken from a finite set of symbols. These symbols are constructed as closely as possible to reality.

Type of actions	a_i-actions	Associated symbol
$\mathcal{W}$-action	Add	+
$\mathcal{W}$-action	Remove	–
$\mathcal{T}$-action	Search	➦
$\mathcal{S}_O$-action	Point	↩
$\mathcal{S}_O$-action	Chat	💬

Table 1: Examples of encoding

Data retranscription method

The aim here is to transcribe the corpus of audiovisual data as faithfully as possible according to the actions ($a_i \in \mathbb{A}$) performed on the objects ($o_j \in \mathbb{O}$) by the subjects ($s_k \in \mathbb{S}$)[2]. The local activity of an individual acting is characterized by a triplet $(a_i, o_{j,}\ s_k) \in \mathbb{A} \times \mathbb{O} \times \mathbb{S}$. Each triplet is a "micro-activity" that can be used to characterize each action performed on objects by the subjects observed. In addition, each micro-activity is ordered by an "occurrence-time marker" seen as a couple mentioning the discretized occurrence (occ) of the latter and the instant t at which it is performed during the audiovisual recording.

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[2] $\mathbb{A}$ is the set of the elementary actions chosen, $\mathbb{O}$ is the set of the objects and $\mathbb{S}$ is the set of the subjects of the experiment.

$$\cdots \xrightarrow[occ]{t} (a_i, o_j, s_k) \xrightarrow[occ+1]{t'} (a_p, o_m, s_k) \xrightarrow[occ+2]{t''} \cdots$$

Between each occurrence-time marker there may be one or more micro-activities starting at the same time. For example, between two clearly identifiable micro-activities, there are several micro-activities to which we are unable to give an order, since we do not know precisely when they were carried out. In fact, we assume that they all start at the same time. In fact, it is also possible for several micro-activities to be located between two markers when they are carried out in an almost immediate period of time.

$$\cdots \xrightarrow[occ]{t} \overbrace{\{\text{List of simultaneous actions.}\}}^{\text{Phase}} \xrightarrow[occ+1]{t'} \cdots$$

Below is an example of the retranscription of micro-activities from an audiovisual recording. Two people try to tile a discrete grid of the size 5×5 with a hole (1×1) using dominoes of the 1×2 or 2×1.

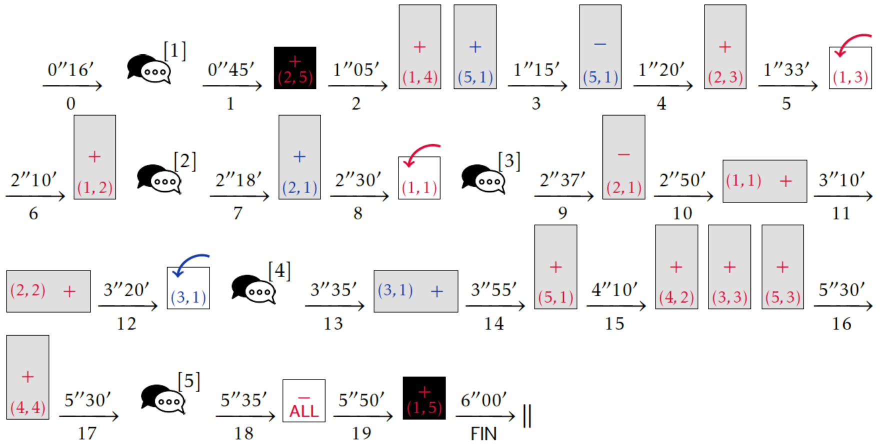

Figure 1: An example of retranscription with two subjects s_1 and s_2 (see Da Ronch, 2022, p. 67)

The "summary": an "enriched" photograph of the Work Zone

We then suggest enriching our transcription with a "summary", *i.e.* an "enriched" photograph of the $\mathcal{W}_Z$ at a given moment. This gives a static view of the situation at a given moment. This summary provides a snapshot of the reality of the subjects at a given moment. It also lets you know exactly how many times the subject's $\mathcal{W}$-actions have occurred, knowing how many times the $\mathcal{W}$-actions of other subjects have occurred. In this summary, we do not consider the occurrence of actions of other types, or time. For instance, we can see that s_1 puts the black hole in coordinate (2,5) on the grid of the size 5×5. This is the first $\mathcal{W}$-action of s_1 given that s_2 has not yet taken any $\mathcal{W}$-action (see e.g., Da Ronch, 2022).

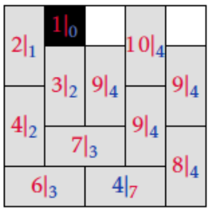

Figure 2: An example of summary in connection with the retranscription of Figure 1 (see Da Ronch, 2022, p. 67)

DESCRIPTION OF THE ORA-MODEL: DATA ANALISIS

The retanscription represents a word of the form $a_i o_j$ which also makes it possible to know, for a transcribed action, who carried it out. We are looking for sub-words in the $a_i o_j$ retranscription sequence relating to the strategies identified in the choice of observables. We then check that they are identical or "close" to these strategies in order to infer that the subjects used a particular strategy that required them to mobilize knowledge related to the practice of a mathematical activity (necessary condition, sufficient condition, reasoning, *etc*.). For an analysis of the transcription of figure 1, see Da Ronch (2022, pp. 71–73).

CONCLUSION ON THE ORA-MODEL

We have presented a model called **ORA** (**O**bservable-**R**etranscription-**A**nalysis) which makes it possible to describe, identify and objectify the traces of the activity of problem-solving subjects by pointing out precisely the identification and interpretation phase. This model also shows its relevance when it comes to solving problems using tangible objects, whose actions are like those carried out in "paper and pencil" form. Discrete mathematics, for example, would seem to be an ideal field, since many mathematical objects (graphs, tiles, polyominoes, etc.), linked to problems of combinatorics, discrete optimization, etc., can easily be represented as tangible artefacts. Finally, the **ORA**-Model seems to offer interesting perspectives in the field of artificial intelligence, as it would allow the collection of important data for a significant number of users and over a potentially high duration (see e.g., kTBS4LA platform, TWEAK team: Cadaso et al., 2017). This model does, however, raise questions about its operationality in the case of abstract mathematical objects that cannot take on a tangible format. As a result, the encoding procedure is much more complex and reading the retranscription is more difficult.

References

Brousseau, G. (1997). *Theory of didactical situations in mathematics* – 1970-1990. Dordrecht: Kluwer. https://doi.org/10.1007/0-306-47211-2

Casado, R., Guin, N., Champin, P. A., & Lefevre, M. (2017). KTBS4LA: une plateforme d'analyse de traces fondée sur une modélisation sémantique des traces. In *Méthodologies et outils pour le recueil, l'analyse et la visualisation des traces d'interaction-ORPHEE-RDV*. https://hal.science/hal-01637548/

Da Ronch, M. (2022). Pratique de l'activité mathématique en médiation : modèles didactiques et conception d'ingénieries [Pratice of mathematical activity in mediation: didactics model and engineering design], PhD Thesis, Université Grenoble Alpes. https://theses.hal.science/tel-04089443

Da Ronch, M., Gandit, M., & Gravier. S. (2023). Modèle pour repérer l'activité de sujets en résolution de problème. In Vandebrouck, F., & Gardes M-L. (Eds.). (2023). *Nouvelles perspectives en didactique des mathématiques : la preuve, la modélisation et les technologies numériques. Actes de la XXIe école d'été de didactique des mathématiques, Saint Marie De Ré, 18 au 24 Octobre 2021*, *Vol. 2* (pp.69–78), IREM de Paris, 978-2-86612-404-5. https://hal.science/hal-03416749