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Epistemological Analysis and Design of Research Situations in Discrete Mathematics: Method and Examples

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Introduction

- **T**heory of **D**idactical **S**ituations is one of the important references in French Didactics (Brousseau, 1997)
- **TDS** and its concepts can be used to design and analyse of teaching and learning situations in different fields of mathematics (e.g., Discrete Math: *Maths à Modeler* Team at Grenoble Alps University)
- **D**idactical **E**ngineering is often used as a research methodology in the French-speaking community for designing and analysing didactical situation in mathematics (Artigue, 2014)

Didactical Engeering

- **DE** involves several stages:

  "preliminary analysis, conception and *a priori* analysis, realization, observation and data collection, *a posteriori* analysis and validation" (Artigue, 2015, p. 471).

- In France, several studies in discrete mathematics education use **DE** as research methodology (e.g., Da Ronch, 2022, Giroud, 2012; Godot, 2005; Ouvrier-Buffet, 2003, *etc.*)
- The epistemological (mathematical) analysis is very important in the preliminary analysis
- This analysis make it possible "to support the search for mathematical situations representative of the knowledge […]" (Artigue, 2014, p. 472).

Research Situation for the Classroom based on TDS and DE Methodology for designing Situations in Discrete Math.

RSC has several didactical conditions (Grenier & Payan, 2003; Ouvrier-Buffet & Gravier, 2022)

- The problem is based on mathematical research
- The mathematical problem should be easily understood for the students
- There are initial strategies with no prerequisites
- An answered question opens a new question
- There exist research variables
- There are a variety of strategies for progressing research and many developments are possible

Remark and Research Question

- No method is specified for carrying out an epistemological analysis in order to determine consistent mathematical situations that are representative of the aimed knowledge and know-how (Da Ronch & Gravier, 2024)

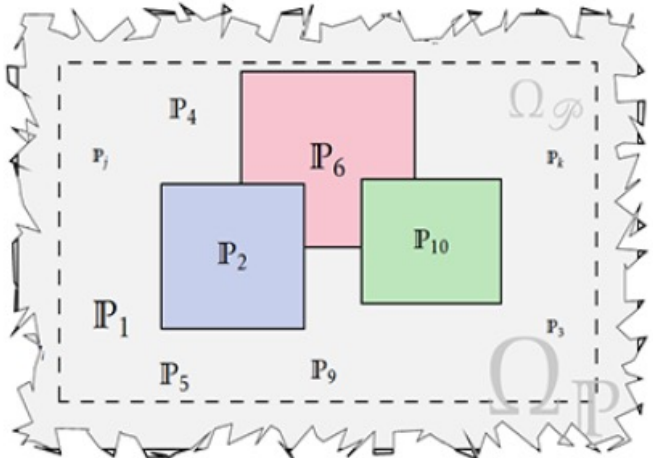
- **RQ:** *How can we carry out an epistemological (mathematical) analysis of a problem to identify mathematical situations that are relevant in the field of discrete mathematics for instance ?*

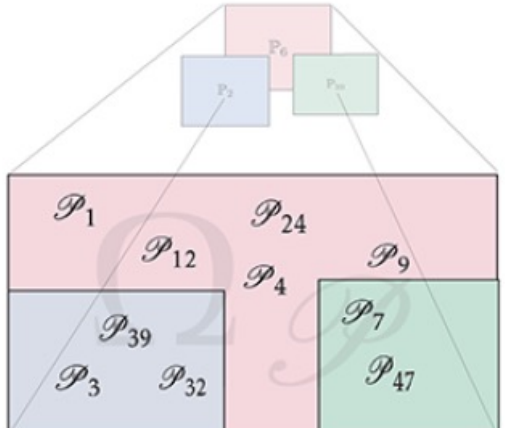
Theoretical Elements for our Method of Epistemological analysis

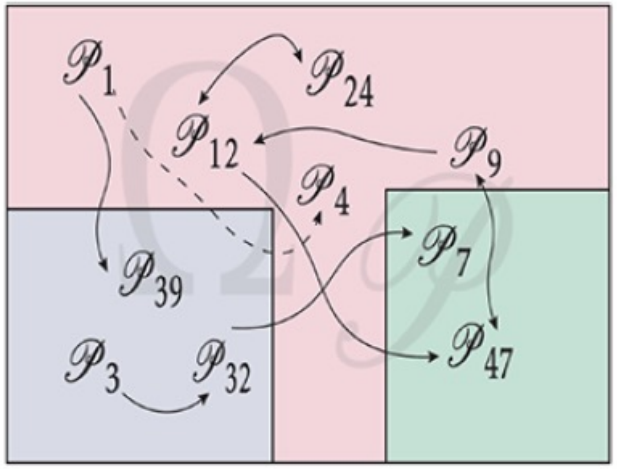
- A mathematical problem is based on 2 aspects (Da Ronch, 2022)
  - Syntatic : A mathematical problem must be formulated as a set of instances and a general question (Garey & Johnson, 1979).
  - Sémantic: Based on the *concept of problem* described by Giroud (2011) and in particular the notion of *problem-space* $\Omega_{\mathcal{P}}$ developed by Da Ronch (2022).
- For the last point : *Significant epistemological quantity* of a problem $\mathcal{P}$ (Da Ronch, 2022)
- *Zoom Concept* (Da Ronch, 2022) : enable to focus on the Problem-Space $\Omega_{\mathcal{P}}$, at different levels of granularity, depending on the target audience and knowledge and know-how aimed through the mathematical situations

Method for carring out an Epistemogical Analysis

- Based on a research problem: identify the sets of problems to which the problem relates
- With the zoom concept, at a finer level of granularity, point out the problems inherent in these sets: design of the Problem-Space $\Omega_{\mathcal{P}}$
- Identify the proximity of these problems using the neighbourhood relationships between problems given by the epistemological analysis
  - Partial sufficiency relationship ($\mathcal{P}_1 \dashrightarrow \mathcal{P}_2$)
  - Sufficiency relationship ($\mathcal{P}_1 \longrightarrow \mathcal{P}_2$)
  - Equivalent Relationship ($\mathcal{P}_1 \longleftrightarrow \mathcal{P}_2$)





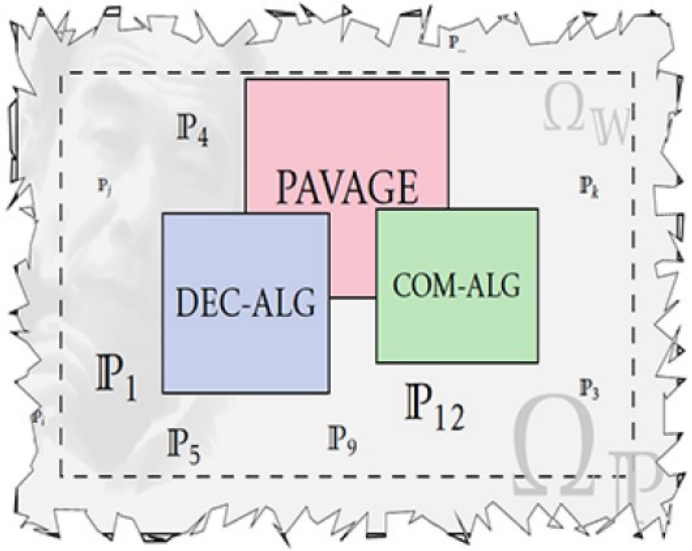


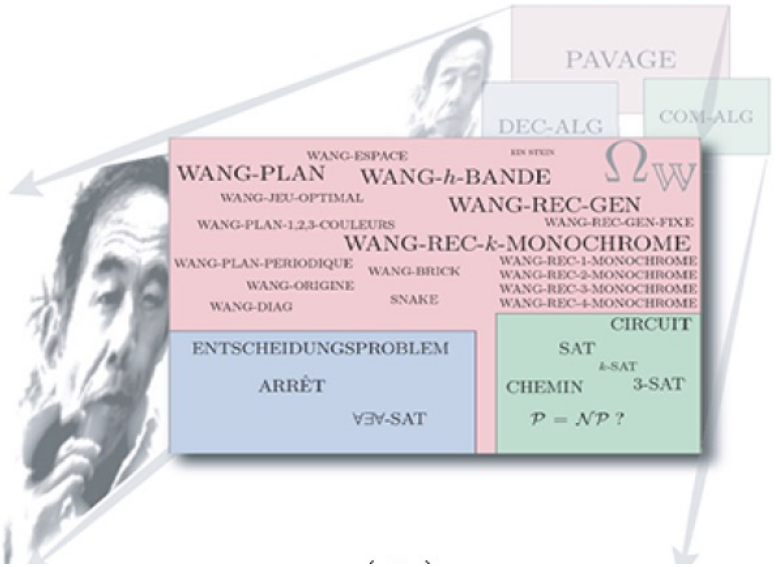
An Example from a Research Problem in Discrete Mathematics : The Domino Problem (WANG-PLANE)

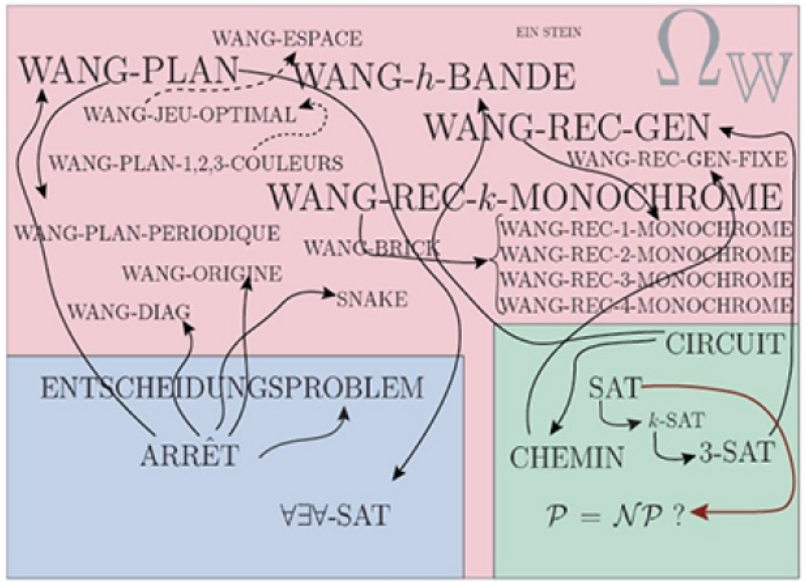
- **Instance**: A finite set of Wang tiles.
- **Question**: Is it possible to tile the discrete plane $\mathbb{Z}^2$with this tileset? (Wang, 1961)

First Zoom on the Space-Problem $\Omega_{\mathbb{W}}$

- This problem is close to 3 problem sets (TILING, COMPUTABILITY and ALGORITHMIC COMPLEXITY)
- Within this space, there are several problems that are linked by relationships (Da Ronch, 2022)

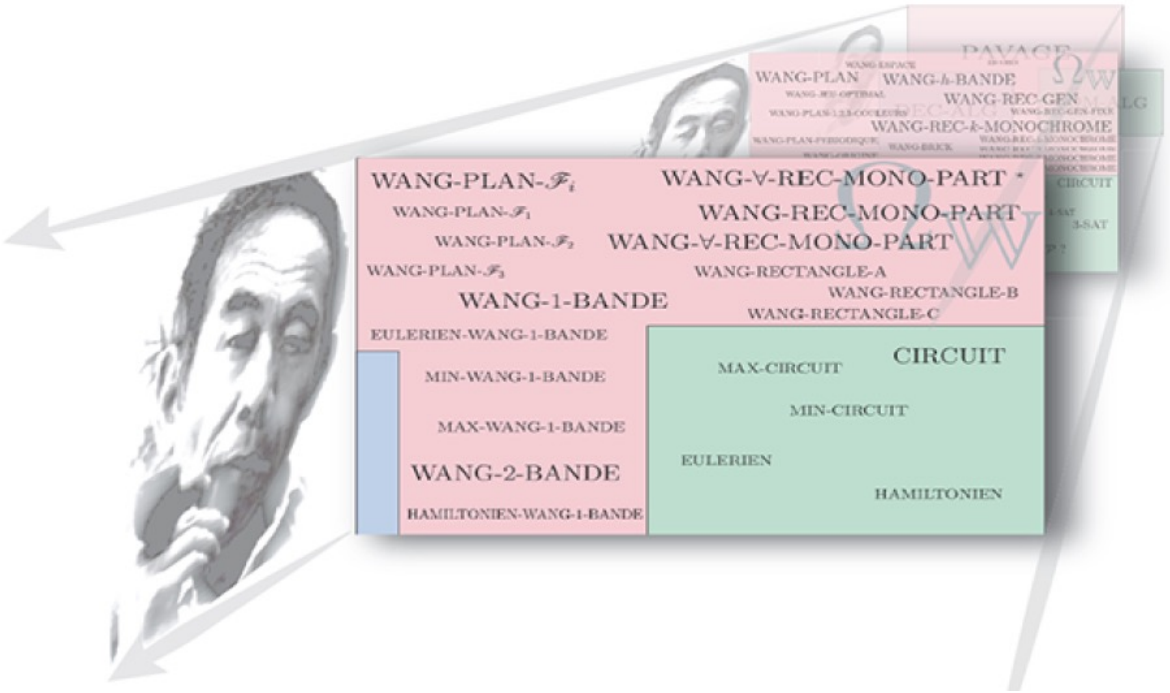


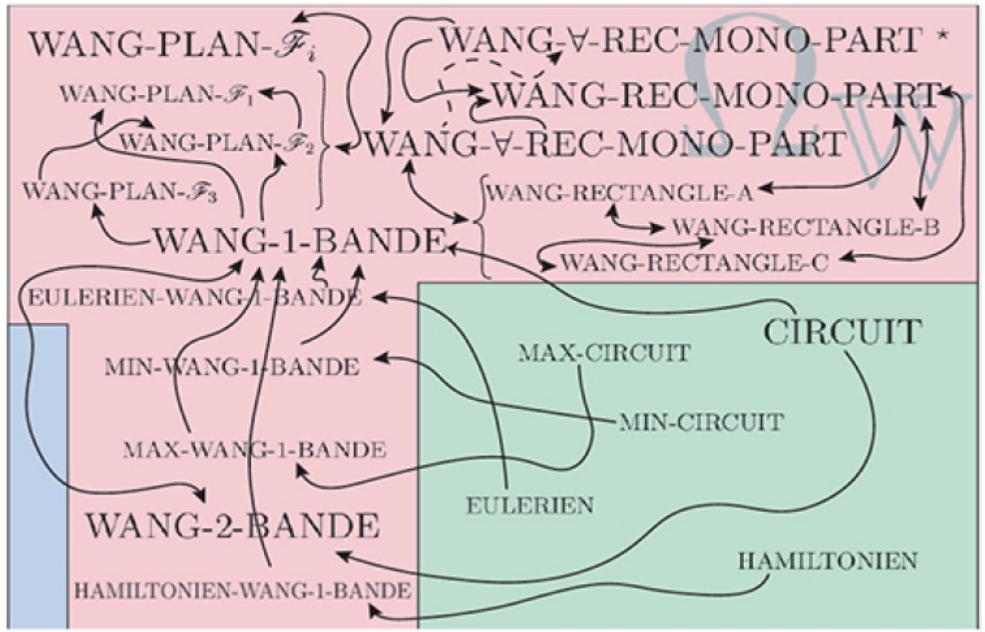




Second Zoom on the Problem-Space $\Omega_{\mathbb{W}}$

- We find other problems with more specific conditions (instances) on the problems (Da Ronch, 2022).





Concepts of Discrete Mathematics mobilised depending on the Number of Zoom

- **1st Zoom**
  - Computabilty notions
  - First-order logic notions
  - Algorithmic complexity notions (NP-Complete, P, PSPACE...)
  - Graph problems (pathfinding problem, cycle finding problem, Depth-First Search algorithm,…)
  - Path Algebra (Binary Matrix, Calculation of the nth power…)
- **2nd Zoom**
  - Arithmetics notions : congruence, Euclidean division, induction principle
  - Geometric notions : translation, periodicity, tiling
  - Algorithmic notions : loop, variable, condition
  - Know-how : Necessary condition/sufficient condition, proof of existence, proof of impossibility, several reasonings (implication, induction, contradiction…)

Conclusion and Research Prospects

- Proposal of a method for carrying out the epistemological study in order to design mathematical situations for targeted knowledge and kwow-how
- The notion of *significant epistemological quantity* still need to be clarified
- Possibility to define by extension an metric that can be used to determine the proximity (or distance) between the mathematical problems (Da Ronch, 2022; Da Ronch & Gravier, 2024)

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