Didactical engineering: an approach for carrying out an epistemological analysis from research problems in mathematics

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INTRODUCTION AND RESEARCH QUESTION

The Theory of Didactical Situations is one of the important references in French didactics (Brousseau, 1997). This theoretical framework can be used to construct of teaching and learning situations, with a focus on didactical engineering as a research methodology (Artigue, 2015). This approach involves several stages: "preliminary analysis, conception and *a priori* analysis, realization, observation and data collection, *a posteriori* analysis and validation" (Artigue, 2015, p. 471). She emphasizes the importance of an epistemological analysis in the preliminary analysis "to support the search for mathematical situations representative of the knowledge [...]" (p. 472). However, no method is specified for carrying out this type of epistemological analysis to determine consistent mathematical situations that are representative of the aimed knowledge and know-how. Therefore, *how can we carry out an epistemological (mathematical) analysis of a problem to identify mathematical situations that are relevant for university students?* To that end, we propose a method of epistemological analysis for designing didactical engineering for learning and teaching mathematics at university. The poster will illustrate this method using a discrete mathematics problem.

THEORITICAL FRAMEWORK

We define the concept of problem from the *syntactic* and *semantic* aspects (Da Ronch, 2022). For the syntactic aspect, a mathematical problem must be formulated as a set of instances and a general question (Garey & Johnson, 1979). For the semantics aspect, we are based on the concept of problem described by Giroud (2011) and in particular the notion of problem-space or universe of problems to characterise the scope of a given problem, and to study its ramifications and its proximity to other underlying mathematical problems by modifying the values of its instances and/or the scope of its question (e.g., Da Ronch, 2022). This will enable us to determine whether the problem holds a significant *epistemological quantity*. This quantity will be judged to be all the more significant if its problem-space or universe contains a significant number of problems in its neighbourhood, and that these problems are linked by relationships (partial sufficiency relationship, sufficiency relationship, necessary relationship and equivalent relationship between problems), based on the proximity of the questions, instances and also the invariants of the proofs used to solve them. The *zoom* concept will allow us to look at this space at different levels of granularity (Da Ronch, 2022).

METHOD FOR CARRYING OUT AN ESPISTEMOLOGICAL ANALYSIS BASED ON A MATHEMATICAL RESEARCH PROBLEM

We describe the universe of mathematical problems as an infinite space composed of problem sets such as $\mathbf{\Omega}_{\mathbb{P}} \coloneqq \{\mathbb{P}_1, \mathbb{P}_2, \dots, \mathbb{P}_i, \dots, \mathbb{P}_j, \dots\}$. Each of these problem sets falls into different branches of mathematics. Thus, during the epistemological analysis of a given problem $\boldsymbol{\mathcal{P}}$, when we wish to identify whether this problem is semantically interesting (not isolated), its universe $\mathbf{\Omega}_{\boldsymbol{\mathcal{P}}}$ is initially limited to sets of neighbouring problems $\mathbb{P}$ to which $\boldsymbol{\mathcal{P}}$ is related by neighbourhood relationships (linked to the question, instances and invariants of the proofs). Here, "neighbourhood" defines, by extension, a "metric" that can be used to determine the proximity (or distance) between problems. Thus, we need to focus on some of these problem sets that are judged to be significantly close to $\boldsymbol{\mathcal{P}}$. In this poster, we will illustrate our points with a contemporary research problem in discrete mathematics: the *Domino Problem* (e.g., Da Ronch, 2022). This problem allows to work on different concepts as the decidability in the algorithmic sense (computability), algorithmic complexity, finding of paths and circuits in a digraph $\boldsymbol{G}$, *etc*. Once these problem sets have been determined, we use the notion of zoom, which allows us to examine the problems in the universe $\mathbf{\Omega}_{\boldsymbol{\mathcal{P}}}$ that belong to the problem sets on which we have focused with an enlargement factor. The choice of problems from these sets is always determined by the epistemological study of $\boldsymbol{\mathcal{P}}$, which makes it possible to establish neighbourhood relationships between the problems of $\mathbf{\Omega}_{\boldsymbol{\mathcal{P}}}$**,** thus giving meaning to $\boldsymbol{\mathcal{P}}$. Thus, the richer the universe $\mathbf{\Omega}_{\boldsymbol{\mathcal{P}}}$ of problems with neighbourhood relationships, the more $\boldsymbol{\mathcal{P}}$ is a semantically interesting problem to study, since it is not isolated, and which, moreover, has a significant epistemological quantity. The epistemological analysis of the problem may be refinement according to the objectives and the target audience. In this way, the process of zoom can be carried out as many times as necessary, depending on requirements.

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