
What Use of Analysis a priori by Pre-Service Teachers in Space Structuring Activities?

15.1. Introduction – an institutional challenge of transposing didactic knowledge

Among the professional gestures expected of teachers is the ability to anticipate the different procedures that can be used by pupils and to identify the knowledge contained in the tasks proposed to them. It is undoubtedly with regard to these aims that a priori analysis figures in the content of most Francophone teacher training courses. However, beyond mere planning and anticipation, the analysis of actions such as intervention (scaffolding) with pupils is also expected. This requires, through training, the construction of systems that suggest an implicit recourse to a priori analysis can be mobilized for this. It is the strength of this implication and the impact of the underlying disciplinary knowledge that we propose to study here. Therefore, after conducting a study on the transposition of the a priori analysis as a professional gesture, we will question the mobilization of the latter by young teachers in French-speaking Switzerland, in training within the framework of an activity aimed at structuring space for young pupils (3–4 years old).

15.1.1. Choice of external transposition: institutional constraints

In Switzerland, education and the school system as a whole are a cantonal responsibility. The same is true for teacher training. Each (cantonal) training institute is free to choose its own training content, provided that it respects very

broad constraints resulting from inter-cantonal agreements. Therefore, it is the trainers themselves, with regard to institutional constraints and their conceptions of the teaching profession, who take charge of the external transposition of the didactic knowledge that will be worked upon in training.

Constraints inherent to the reference works

Although these inter-cantonal agreements do not propose standardized study or training plans, they nevertheless lead to the common development of unique pedagogical resources (*Moyens d'Enseignement Romands*, MER), the use of which becomes de facto mandatory for all teachers and schooling. With regard to mathematics resources in elementary school, the editors of the latest edition of the MER, introduced in 2018, advocate in the general comments accompanying the activities, an explicit use of a priori analysis:

It will therefore be necessary to determine the conditions for this transition [in this case, an informational leap] to be made indispensable. To do this, it will be necessary to carry out an *a priori* analysis of the activities proposed to the pupils (see text on *a priori* analysis) and identify the choices to be made (didactic variables) to force this transition (MER, n.d. extracted from *Le Nombre – Premier Apprentissage – Cycle 1*, translated by author)

Of course, the compulsory nature of the use of these resources will have an impact on the choices of external transposition for the content of different training institutes: these will have, from then on, an almost explicit mandate to teach a priori analysis, questioning the nature of this *knowledge to be taught* and of the possible distinctions to be made with a *scholarly knowledge*, which would remain to be determined¹.

Constraints inherent to the training institute

Like several Swiss institutes, the Haute Ecole Pédagogique du Valais (HEP VS) chose to train (and evaluate) teachers by professional competencies in 2013, which necessitated a certain “linking” of course and practicum content in the pupil’s training path.

Among the five Francophone training institutes, our interest in the HEP VS is motivated by the explicit appearance of a priori analysis in the practicum criteria (the Francophone version of which is reproduced in Table 15.1): it is therefore

¹ The bibliography presented by the MERs does not provide references or an understanding of the term “analysis a priori”.

possible to infer that it must be included in the content of the various courses offered².

Specific criteria	Evaluation
SC 1	Clarify the object of study through the analyses (conceptual and a priori) and delimit it in order to promote its accessibility (identification of difficulties in making choices).
SC 2	Design and plan in writing a teaching/learning device taking into account the points raised in the analyses (of conditions, conceptual, a priori).
SC 3	Accompany pupils in their learning. Justify his/her interventions in light of teaching/learning theories.
SC 4	On the basis of the planned objective, anticipate and set up a formative evaluation system: analyze the pupils' oral or written traces in order to propose regulations to support learning.

Table 15.1. Evaluation criteria for the 3rd-semester internship

The titles of the different criteria tell us that, in addition to planning (SC 1 and SC 2), future teachers will be evaluated – during their third-semester internship of a training program that includes six semesters – on the relevance of their interventions and on their ability to justify them (SC 3). The same applies to the creation of a formative evaluation system (SC 4). Therefore, beyond a single planning technique, this a priori analysis, seen as knowledge to be taught in training, should also, despite its implicit nature in the formulation of SC 3 and SC 4, be able to be mobilized in order to justify the accompaniment of pupils and the setting up of a formative evaluation system. It is therefore up to the trainers to ensure the transposition (Chevallard 1985) of this didactic knowledge from Semester 3 onwards, both in terms of techniques and with the aim of bringing out its different applications, or to use the terms developed later, the *work of organization*, in the sense of Chevallard (2002). It is on the basis of this field of applications that we will circumscribe this a priori analysis – constituting a *knowledge to be taught* – characterized as a professional gesture and which will be distinguished from that thematized by Artigue (1990a).

15.2. Theoretical framework

This first distinction, inherent to external transposition (Chevallard 1985), leads us to describe, in this section, the internal transposition of this knowledge. As

² At this institution, didactic courses in mathematics are located in Semesters 3 and 4.

indicated above, the didactic modules of mathematics are located in Semesters 3 and 4 of a six-semester program. With regard to institutional requirements (evaluation during Semester 3), the a priori analysis is addressed from the first sessions of the first module. The team of trainers in charge of these modules has chosen to deepen numerical notions in Semester 3 and geometrical/spatial notions in Semester 4.

Choice of internal transposition: the moments of the study of the analysis a priori

In the desire to inscribe within its training system a paradigm of questioning the world (Bosch et al. 2018, cited by Candy 2020), the team of trainers chose to simulate professional practice in training. It thus based itself on both a role-playing device (Lajoie and Pallascio 2001) and on “the moments of the study” (Chevallard 2002), considering the a priori analysis as knowledge to be taught generating a punctual Mathematical Organization³. This overlap generates some adaptations to the training device initially set up by Lajoie and Pallascio (2001). Therefore, for a task T , consisting of “carrying out an a priori analysis”, we find in the planning of Semester 3 the following progression, foreseen to take place over approximately six 2-hour long sessions:

– *moment of the (first) encounter with task T* : once the whole “role-playing” device has been presented (everyone knows that an intervention and a debate will follow), the pupils are led, in groups, to solve a problem in several different ways, without revealing their processes to the other groups;

– *moment of the exploration of T and the emergence of technique t* : a pupil who will play the role of the teacher is randomly drawn from one of the groups, which have also been randomly selected. The other groups each send a representative to play a pupil. The “teacher” has to react to the productions presented to them (which they had not necessarily anticipated). A collective debate follows the intervention. During the debate, the trainer opts for a listening posture (without intervening) and leads the pupils to comment and justify the interventions they made⁴;

– *moment of the construction of the technological-theoretical block*: extraction from the trainer of the elements of debate that can constitute technological elements;

³ These elements are presented here in a very descriptive way and are not intended to justify the theoretical choices and analogies made by the trainers, which will undoubtedly become a topic for future publications.

⁴ During this stage, the students activate spontaneously, with regard to the position of the trainer, elements such as the importance of epistemological analysis in the context of numerical activities. This position is justified by the goal for trainers to fit into a paradigm of “questioning the world”.

– *moment of the institutionalization*: carried out by the trainer who leans on the elements of debate;

– *moment of the work of the mathematical organization (and in particular of the technique)*: application on other types of situations, in particular for purposes of support and the creation of an evaluation device.

15.3. Research questions

Considering the succession of disciplinary contents dealt with over the course of the semesters, we will use these institutional configurations to verbalize the following research questions. Is a training device that recreates the terrain (role-playing) and the didactic indications proposed to the teachers in training by the various institutions sufficient for the appropriation and mobilization of a priori analysis (as defined at the end of the previous section) within the framework of space structuring activities? Still in such a training device (role-playing game), what are the characteristic elements of a priori analysis reinvested during the preparation and piloting of space structuring activities, by pupils who have previously worked during their training on the different applications of this professional gesture in a numerical domain? Is an epistemological analysis of the objects manipulated at the level of knowledge carried out by the students in training within a framework of space structuring? Can a notable difference be noted with activities of a numerical nature?

15.4. Methodology

In order to allow for the comparison aimed at through our second research question, this training device is based, at the beginning of Semester 3, on a numerical activity which will be described in the following section.

At the beginning of Semester 4, a similar device is reproduced with the same pupils (admittedly, without being able to speak then *stricto sensu* of a *first encounter*), this time based on an activity of structuring space.

Therefore, our body of data focuses each time on recorded discourse (debate described at the time of *T's exploration* – hereafter Phase 2) and written traces (produced at the time of the *first encounter* – hereafter Phase 1) gathered during two successive academic years (2018–2020), each cohort of future teachers comprising approximately 30 pupils.

As far as our first research question is concerned, we refer to Artigue (1990a) in order to characterize the possible mobilization of the a priori analysis (as defined in

Section 15.1.1), which can be observed with regard to the modulation, during the piloting of the activity (Phase 2), of the identified didactic variables (Phase 1) or of various mentions during the debate (Phase 2). The same applies to the second research question: it is also in these traces (Phases 1 and 2) that we extract possible questionings or epistemological studies relating to the hosted knowledge.

Selection of activities and brief analysis

The numerical activity is chosen (Semester 3), in view of the plurality of available resolution techniques – themselves resulting from various technologies – and the different ostensives that can be mobilized (correspondence table, linear equation, etc.). The trainers chose a situation (Figure 15.1) that stated the mobilization of the notion of proportionality⁵.

5. The Politics of Small Steps

To cover the 36 meters of the perimeter of her rectangular pool, whose length is twice the width, Paulette counted 54 steps.

How many steps did it take her to cover its width alone?

Figure 15.1. Excerpt from *MER 8H* (pupils aged 12)

In order to test the possible (re)mobilization of the a priori analysis (see the first research question), it was then proposed to the pupils, during one of the first sessions of the following semester, the situation *Une Chaise pour deux* (A Chair for Two), extracted from the *Axe Thématique Espace des Moyens d'Enseignement* (1-2H, pupils of 4–5 years old):

– Time 1 (real-time imitation): when entering the room, the pupils, in pairs, orient their chairs in the same direction so that they can turn around without touching the other chairs; “You take turns making up a statue using the chair. The other imitates the statue and the first validates or corrects it”;

– Time 2 (sculpture by instructions): three roles are proposed: the model, the sculptor and the sculpture (a screen separates the model from the sculpture). The model: “Invent a statue using the chair.” Sculptor: “Tell the sculpture what to do to reproduce the model.” To the sculpture: “Do what the sculptor tells you.”

⁵ This is at least the objective attributed to it in the comments accompanying the statement. An a priori analysis, difficult to develop here, would however show that this can be discussed.

Located at the beginning of a teaching sequence, this activity, whose stated objective is to lead the pupil to “Determine his/her position or that of an object according to different reference points”, is presented as an “introductory activity” with the intention of “helping the pupil to discover (construct) the knowledge taught”. Its place upstream of a sequence and the intention announced by the authors of the MER, would justify that the teacher carries out, during the piloting, some modulations of the numerous didactic variables, all the more so given that the didactic environment thus constructed proves to be sufficiently antagonistic so as to issue to the “sculptor pupil” with strong feedback on his/her production.

One of the interests of this activity lies in the fact that, in addition to its statement, the didactic variables are not listed exhaustively by the MERs. Only those related to the types of indications provided or the number of consultations allowed are implicitly mentioned in the comments:

The activity can be conducted without verbalization but in the form of modeling or sculpture. The sculptor must reproduce the proposed model by manipulating the sculpture. It is possible to limit the number of times the model is consulted. The validation is done by putting the model and the sculpture together. In this case, it is only at this point that verbalization is required (MER, n.d., comments on the “A Chair for Two” activity, *translated by author*).



Figure 15.2. Training situation where the pupil playing the role of the teacher (scarf) intervenes with the pupil-sculptor (red sweater), in charge of giving instructions to the statue (striped sweater)

The spatial knowledge mobilized differs according to the respective orientation of the chairs – while satisfying the instruction, they can be arranged in line with each other (respecting the “same orientation” constraint) and thus promote elements

inherent in translation and not in symmetry, as suggested by the arrangement shown in Figure 15.2 – or of the different protagonists.

Therefore, following the example of the numerical activity presented in the previous semester, several resolution procedures that call upon different types of identification can be mobilized (recourse to bodily laterality, internal and/or external identification of the situation, cardinal points, etc.), which can then generate questions of an epistemological nature as to the hosted knowledge.

15.5. Results

First of all, we note that, in the case of the numerical activity, even if the importance given to each of the parts varies, all the debates (Phase 2) present a relatively similar structure: they are initiated by questioning the validity and the status of the techniques mobilized (“But are we allowed to do that?”, “Is it on the syllabus?”, etc.) before questioning different epistemological components (“But what is proportionality apart from a [cross-reference] table?”) ~~before questioning different epistemological components (“But besides, what is proportionality apart from a [cross-reference] table?”)~~ and ending on the interest of considering the elements developed during the a priori analysis in order to build an evaluation device (“Ah, but that’s how you would make your evaluation criteria!?”), etc. In general, the conclusion of the three debates points to the need to carry out an epistemological analysis of the objects handled, in order to be able to pilot the activity a little more easily.

Finally, a significant proportion of the traces produced during the preparation (Phase 1) testifies to a search for elements of the statement which are, moreover, modified by the teachers during the various interventions (Phase 2). As an example, we point out a large panel of ostensives and numerical values.

On the other hand, in the case of the spatial activity, none of the traces produced during the preparation (Phase 1) seem to point to elements that can then be modulated by the teacher. All the pupils who took on the role of the teacher opted for an intervention centered on the validity of the production (“Do you have the impression that you have done it right?”, “So, it’s not right”) and on the reproduction of the model’s position (“If I put myself like this and if I turn around, do I have the same thing?”). In the vast majority of cases, the spatial vocabulary was not mobilized during the intervention (Phase 2), neither by the pupil nor by the teacher (“You put the other hand on the chair and you raise the other arm”). The various discussions (from which no common structure seems to emerge) do not propose any links between the preparation and the piloting; none of them raises the importance of an epistemological analysis.

15.6. Conclusion

We thus note several differences between numerical and spatial activities.

In the first case, while the a priori analysis is still unknown to them, the pupils tend to look for several procedures during the preparation, and leave themselves the latitude to modify various elements of the statement during their interventions. In a certain sense, an a priori analysis is carried out en acte by the group during the exchanges (Phase 2) at the end of which different applications are spontaneously pointed out (then taken up again later, see Phase 5). At a more advanced stage of their training, however, the trainee teachers have difficulty anticipating various procedures, identifying didactic variables and mobilizing them during the piloting of spatial activities. Moreover, no recourse is made to the various didactic concepts stabilized beforehand in the training, during the discussions on the spatial activity. It is the same for the epistemological analysis, however carried out beforehand in a numerical framework and then considered necessary.

We hypothesize strongly that, unlike notions of proportionality, the self-evident nature of spatial notions prevents trainee teachers from “detaching themselves from the illusion of transparency” (as noted by Artigue 1990b, p. 245, *translated by author*).

Therefore, we conclude that, contrary to what seems to prevail in the context of numerical activities, the training device set up in the training does not allow trainee teachers to appropriate a priori analysis as a professional tool for space structuring activities, at least in meso-space: despite the numerous didactic variables that can be mobilized, it is not perceived as having an impact on scaffolding in the session or on professional interventions in general.

15.7. References

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